Development of Soft Mathematical Morphology and its role in Image Processing - A Review Article

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Abstract: Image processing was started by NASA for war applications, during Second World War. After that, image processing was found applications in so many areas. At the same time the development of computer was started due to semiconductor technology development, I.C. technology development and size is reduced, speed is increased, and memory is increased and rate is reduced. Due to this scientists were attracted by these computers and started work in various areas, with computers. A set of scientists were attracted by image processing and they started research in various areas like noise elimination, image smoothing, edge enhancement, etc. areas. At the same time Serra and Matheron were asked to study characteristics of ores. Then mathematical morphology was born and with the help of this they studied mining properties. But later on it was found that this mathematical morphology is having applications in image processing and they are functioning efficiently. So mathematical morphology entered in to image processing and scientists started working in this area and they developed mathematical morphology to a peak level. But this mathematical morphology is suffering with its rigidness in the definition. So changing the definition and introducing flexibility in definition, soft mathematical morphology is introduced, and this new definition eliminates the draw backs of mathematical morphology, and provides better outputs. So in this paper I started to discuss soft mathematical morphology and its applications in to various areas because new researchers create interest in this area and they develop this area.

Key Words: Mathematical morphology, Erosion, Dilation, Primitive morphological operation, threshold, Multi scale morphology, open, close, soft erosion, soft dilation, soft open, soft close.

1 INTRODUCTION

In this paper soft mathematical morphology is explained and definitions are given and various researches of this area is introduced in brief such that it will create interest in this area. In this paper soft mathematical morphology is explained. Its extensions are explained. Its applications in smoothing edge enhancement, medical area are explained, structuring elements, structuring element de composition, are explained thinning and thickening are explained. Skeletonization is explained. Multi scale morphology is explained. Iterative morphology is also explained. For each area references are also given.

2 DEFINITIONS:

The primitive morphological operations are dilation and erosion. By means of these operations only, all the remaining morphological operations may be defined. These two morphological operations play the role of bricks, for a house. In soft mathematical morphology, the new morphological definitions are soft erosion, soft dilation, and soft open, soft close. All the operations are given in this section.

2.1. DILATION:

These operations may be defined in so many ways. Different researchers defined this operation in different ways.

Def 1: Let A and B be subjects of \( E^N \) (where N is Space) the dilation of A by B, is denoted by \( A \oplus B \) and is defined by \( A \oplus B = \{C / C = a + b \text{ for some } a \in A \text{ and } b \in B \} \)

Def 2: \( A \oplus B = U (A)_b \)
b ∈ B  Where A is the image and B is the structuring element.
Here (A)_b means, translation of A by b, defined as
(A)_b = {C / C = a + b; a ∈ A}

Def 3:-  (I ⊕ S) [x, y] = 1 if  | I ∩ S' (x, y) | ≥ 1
                                      = 0 otherwise.
Here, I is the image  
S: structuring element
S': reflection of S about the origin
[If S.E. is having origin, at its centre point then S= S'.]
I (x, y) denotes image pixel value at the coordinate (x, y)
|Z| denotes the cardinality of the set Z;
S(x, y): S translated by the displacement (x, y).

2.2. EROSION:

This morphological operation also defined in so many ways, by different researchers.

Def 1):- The erosion of A by B is denoted by A⊖B, and is defined by
A⊖B = {x/x + b ∈ A for every b ∈ B} Here x ∈ EN when EN= N space.

Def 2):- A⊖B = {x/ for every b ∈ B, there exists and a ∈ A, such that x=a-b}

Def 3):- A⊖B = {x/ (B) x ⊆ A}. Here A is image, B is S.E.
Here x ∈ EN (B)_b. Translation of B by “x”

Def4):- A⊖B = ∪ b∈B( A)_b  Here “A” is the image and B is the S.E.
(A)_b : Translation of A by b

Def 5):- (I⊖S) [x, y] = 1 If | I ∩ S(x, y) | ≥ |S|
                                      = 0 otherwise
Here I is image and S is S.E.  I(x, y) denotes image value at coordinate (x, y)
|Z| denotes the cardinality, of the set Z.
S (x,y): S translated by the displacement (x, y)

2.3. OPEN:

It is a composite morphological operation. Open can be defined by the two primitive morphological operations, dilation and erosion operations.

Def:  Open can be defined as, Erosion on the image, followed by Dilation. Open can be represented as, “O” symbol.  B O K means image “B” is opened by the structuring element “K”
B O K = (B⊖K) ⊕ K

2.4. CLOSE:

It is also a composite morphological operation.
Def:- “Close” can be defined as, dilation on the image, followed by, erosion.
Close can be represented as “●” symbol.
B ● K means, image “B” is closed by the S.E., “K”.
B ● K = (B⊕K)⊖K.

2.5. SOFT EROSION:

Soft erosion was defined as (9)
If threshold value= 1 then E(1) may be defined as
(I⊖S(1)) [x, y]= 0 If | I ∩ S(x,y) | ≥1
= 1 other wise.
Here, S(1) means, threshold value=1, in the 3/3 sub image which is chosen, from the image.
If threshold value= 2 then E(2) may be defined as
(I⊖S(2)) [x, y]= 0 If | I ∩ S(x,y) | ≥2
= 1 other wise.

2.6. SOFT DILATION:

Soft dilation was defined as (9)
(I⊕S(m)) [x, y] = 1 If |I ∩ S(x,y) | ≥m
= 0 otherwise.
Here “m” is threshold value where 1 ≤ m ≤ |S|. |S| is the cardinality of S.
Soft Erosion may be defined as
(I⊖S(m)) [x, y] = 0 If |I ∩ S(x,y) | ≥m
= 1 otherwise.  I = inversion of I; m= threshold ≤|S|.
[The exact definition, given in “9”, is slightly modified, according to the requirement, but without changing the meaning. Here the main assumption is origin is at central place of the structuring element and structuring element is assumed to be a square grid.]

2.7 SOFT OPEN:

Open is a composite morphological operation. In this section, open is discussed in soft morphological environment.
Operation on image I, by Structuring Element "S", may be defined as
Open (I , S) = ((I⊕S)⊖S)
Open can be defined as, erosion on the image, followed by dilation. In soft morphological environment, soft open can be defined as, soft erosion on the image, followed by soft dilation. 

So, it can be symbolically represented as, soft open (I, S\textsuperscript{(m,n)}) = ((I ⊖ S\textsuperscript{m}) ⊕ S\textsuperscript{n}).

The m, n will indicate threshold values.

According to above convention, it is understood that, soft erosion on image (I), with threshold value (m) and then perform soft dilation on the soft eroded image, with threshold value “n”. Normally, the following convention will be used, for the representation of soft open.

\[ O\left(\frac{m}{n}\right) \quad OR \quad O(m,n) \]

\[ O(m,n) \] means soft open, with thresholds m & n, soft erode image by threshold value "m" and then soft dilate, the resultant image by threshold value "n".

\[ O(1,2) \] means, soft open, with thresholds 1 & 2. Soft Erode image by threshold value “1”, and then soft dilate, the resultant image by threshold value “2”.

2.8.

2.8. SOFT CLOSE:

Close is a composite morphological operation. In this section, close is discussed in soft morphological environment. 

Close operation, on image “I”, by Structuring Element “S”, may be defined as

Close (I,S) = (I ⊖S) ⊕ S

Close can be defined as, dilation on the image, followed by Erosion.

In soft morphological environment, soft close, can be defined as, soft dilation on the image, followed by soft erosion.

So, it can be symbolically represented as,

\[ C\left(\frac{m}{n}\right) \quad OR \quad C(m,n) \quad (\text{where} \ C(m,n) \text{ means, soft close, with thresholds m & n}) \]

Soft dilate image, by threshold value “m”, and then Erode, the resultant image, by threshold value “n”.

\[ C(1,2) \] means, soft close with thresholds 1&2. Soft dilate image by threshold value “1”, and then soft erode, the resultant image by, threshold value “2”.

3. NEED OF SOFT MORPHOLOGY:

The Soft Morphology is extension to mathematical morphology. Even though mathematical morphological operators are efficient, they suffer with a few drawbacks. In primitive morphological operations, erosion, one or two mismatched pixels of image prevent the structuring element from fitting perfectly. It is the basic morphological operation, quantifies the way in which, the structuring element fits into the image. Erosion is an “All or nothing” transformation, implemented using bitwise “and”. So, erosion will be sensitive to noise.

In primitive morphological operation, dilation, isolated pixels, even though, they are irrelevant to the image’s content, significantly affect the output of the transformation. The net effect is an increased number of large spurious particles, increasing the confusion in the dilated image. So, noise will be added, which may be named as additive noise. (9).

But, many applications require more tolerance to noise than is provided by erosion and dilation. Soft morphological operators possess many of the characteristics, which are desirable, perform better in noisy environments. (9)

So, the soft morphological filters, improve the behavior of standard morphological filters, in noisy environment. The soft morphological filters are better compared to mathematical morphology in small detail preservation and impulse noise. In soft morphology, it preserves details, by adjusting its parameters (23). It can be designed in such a way that, it performs well in removal of salt – and – pepper noise as well as Gaussian noise, simultaneously. (27)

A soft morphological filter can be designed in such a way that, it reduces periodic noise also (32). A filter designed in frequency domain, can function better for smoothing & edge enhancement, according to our requirements. The reason is that by tuning its frequency. But the design involves complex computations. But using soft morphological filters, using very simple computations we can achieve the quality of image processing, to that of filters in frequency domain, which involves complex computations (33). So, we can conclude that soft
morphological filters perform excellent, compared to morphological filters. The idea of soft morphological operations is to relax, the standard morphological definition, a little, in such a way that, a degree of Robustness is achieved. While, most of the desirable properties of standard morphological operations are maintained. The soft morphology was introduced by KOSKINEN etc, and developed by researchers, who are specified in the next coming sections.

4. A FEW IMPORTANT CONCEPTS:

In this section a few back ground concepts relevant to main topic of this paper is given.

4.1. MATHEMATICAL MORPHOLOGY:

Actually J. SERRA (1) and MATHERON (2) are founders of mathematical morphology. They have explained all the fundamentals of mathematical morphology in their books. There are some more composite operations, like thinning, skeletonization etc. in addition to open and close. Any way J. SERRA and MATHERON have developed mathematical morphology for one purpose. But they have entered in to image processing and created wonders. These four operations are discussed thoroughly, with properties and proofs and extensions to gray scale in 3. Really Robert M. Haralick, Stanley R. Sternberg, Xinhua Zhuang have elobarated these morphological operations with examples. Mr. H.J.A.M. HEIJMANS has given a detailed discussion of these operations in A. Till now the light is thrown on the fundamentals of mathematical morphology (1,2,3,4).

The morphological operations are suitable to apply on binary images only. But later these operations are extended to gray scale images also. One method of applying these operations on gray scale images is discussed by PETROS MARAGOS etc. They have (5) proposed a method to convert a gray scale image to binary image series. This method, named as threshold superposition, has opened new doors into this area. Morphological operations may be applied on these binary images, later on, these processed binary images are integrated to get, a processed gray scale image. So, the methodology, proposed by Maragos has extended morphological operations to gray scale environment also. They have discussed the necessary mathematical background, theorems, examples etc.

Actually, applications of morphological operations were extended by SERRA also. Later STERNBERG concentrated in this area. In depth study was done (the theoretical analysis) by J.A.M HEIJMANS (6), in this area. PETROS MARAGOS (7) has discussed about morphology also. PETROS MARAGOS (8) has discussed about morphology and given theoretical analysis.

4.2. ITERATIVE SOFT MORPHOLOGY:

Iterative morphology is a very important concept, which is having wide range of applications, which are going to be explained in the next coming sections especially soft morphology point of view. The applications may be seen in skeletonization, thinning, thickening, structuring element Decomposition, segmentation, etc.

In iterative soft morphological environment, the following convention may be used.

\[(E^{(1)})^2 : \text{Soft Erosion, with threshold value } = 1 \text{ applied, 2 times on the image.}\]
\[(E^{(1)})^5 : \text{Soft Erosion, with threshold value } = 1 \text{ applied, 5 times on the image.}\]
\[(E^{(x)})^y : \text{Soft Erosion, with threshold value } \text{“x”}, \text{ applied “y” times on the image.}\]
\[E^{(1)}, E^{(2)}, E^{(3)} : \text{Soft Erosion, applied with threshold values } 1, 2, 3 \text{ on the image.}\]
\[E^{(1)}, E^{(2)}, E^{(3)} : \text{Soft Erosion, applied with threshold values } 1, 2, 3 \text{ on the image.}\]
\[E^{(x)}, E^{(y)}, E^{(z)} : \text{Soft Erosion, applied with threshold values } x, y, z \text{ on the image.}\]
\[(D^{(1)})^3 : \text{Soft Dilation, with threshold value “1” applied “3” times on the image.}\]
\[(D^{(1)})^4 : \text{Soft Dilation, with threshold value } = 2, \text{ applied “4” times on the image.}\]
\[(E^{(x)})^y : \text{Soft Dilation, with threshold value } = x, \text{ applied “y” times on the image.}\]
\[D^{(1)}, D^{(2)}, D^{(3)} : \text{Soft Dilation, applied with threshold values } 1, 2, 3 \text{ on the image.}\]
\[D^{(x)}, D^{(y)}, D^{(z)} : \text{Soft Dilation, applied with threshold values } x, y, z \text{ on the image.}\]
\[(O (1, 2))^2 : \text{Soft open applied thrice on the image, with thresholds } 1, 2\]
\[\text{[Soft Erosion threshold value } = 1, \text{ Soft Dilation threshold value } = 2]\]
\[(O (x, y))^n : \text{Soft open, applied } \text{“n” times, on the image, with thresholds } x, y]\n\[\text{[Soft Erosion threshold value } = x, \text{ Soft Dilation threshold value } = y]\n\[O(p, q) O(x, y) : \text{Soft Open applied twice on the image, with different thresholds.}\]
O (p, q) O (r, s) O (x, y): Soft open, applied thrice on the image, with different thresholds.

(C (1, 2))_n: Soft close applied four number of times on the image, with Soft dilation threshold value = 1, Soft Erosion threshold value = 2.

[C (1, 2)]_n: Soft close applied “n” number of times, on the image with thresholds 1, 2.

(C(x, y))_n: Soft close applied “n” times, on the image, with thresholds x, y.

C(p, q) C(r, s) C(t, u): Soft close applied on the image, thrice, with different thresholds.

4.3. MULTI SCALE SOFT MORPHOLOGY.

Multi scale morphology has extended its applications to Image Smoothening, Edge Enhancement, Segmentation, Remote Sensing, Radar image analysis, Medical area etc.

It is having special applications, like enhancing weak Edges, Decay analysis of wood, critical analysis of (ECG) Cardio imagery (Identification of critical points), Getting results which are helpful for pilots, lunar landing etc.

3D Structuring Elements in Multi Scale are also designed. Multi Scale Mathematical Morphology is integrated with top – hat training, wavelet training, N.N etc. These concepts are going to be explained in the next coming sections, especially in soft morphology point of view.

4.4 MULTI SCALE ITERATIVE SOFT MORPHOLOGY

If Multi Scale Soft Morphology and iterative soft morphology are combined, a new area Multi Scale iterative Soft Morphology Emerges. [MSISIM]

The research is Neo Natal in this area. A lot of scope for research is there in this area. So, it is necessary to study the properties in this area, which will help, for development of this area. These concepts are going to be explained in the next coming sections.

5. SOFT MATHEMATICAL MORPHOLOGY:

The importance of soft morphology is explained in “section 3”.

So, here in this section concentration is done, in reviewing of the research in this area, MICHAEL A. Z MODA and LOUIS. A. TAMBURINO discussed (9) morphological operations, soft morphological operations in detail. In this paper they discussed the definitions of Erosion, Dilation on the basis of methodology like counting, which is suitable to extend to soft morphological operations, by fixing threshold values. They discussed some more algorithms for implementation of soft morphological operations, properties up to some extent. PAULI KUOSMANERI etc. (10) have discussed about statistical properties of soft morphological ops. They discussed about noise reduction using soft morphological ops, with detail preservation, in this research paper. The above authors discussed in another research paper (11) about the relation in between soft morphology ops as well as stack filters. SHIH, F.Y. etc. discussed (12) soft morphological properties are discussed up to some extent. Some of the properties are stated and idempotency is discussed up to some extent. They discussed about soft morphology op’s in gray scale, using threshold super position theorem. They discussed about implementation of soft morphology op’s, using logic gates also. Any way, it discussed soft morphological operations in a few dimensions.

PU, C.C. discussed about (13) implementation of soft morphological op’s in gray scale. They integrated super position property and stacking to extend soft morphology from binary scale to gray scale.

PAULI KUOSMANER & JAAKKO ASTOLA (14) also discussed, statistical properties, of soft morphology op’s, up to some extent, with connection to stack filters.

GASTERATOS, a discussed (15) a new technique, for the realization of soft morphology op’s basing upon majority gate algorithm system architecture, for implementation of soft morphology op’s, is also presented. MICHAEL A. ZMUDA (16) proposed an algorithm for implementation of soft morphology ops. Normally voting logic also may be used, across neighborhoods, defined by the S.E.

But, in this algorithm instead of processing all the votes, a few votes may be chosen randomly and the service of FSM also, will be taken, in implementation of this algorithm. It is faster than conventional algorithms. Accuracy: more than 90%.

ZHAL CHUNHUI (17) designed soft morphological filter, using genetic algorithm. It is in optimized and improved algorithm. PERTTIT. KOI VISTO, etc. (18) also concentrated and discussed improved algorithms for soft morphological ops, using genetic algorithms.

M. VARDA VOULIA etc. (19) designed algorithms for small detail preservation and impulse noise suppression, using soft morphological op’s [soft
vector morphology] in color environment and shown better results compared to algorithm designed, based on morphological operators. [Mathematical vector morphology]. A. GASTERATOS, etc. (20) discussed about structuring element decomposition, in soft morphological environment. A. GASTERATOS etc. (21) discussed about extension of fuzzy theory into soft morphology. G. LOUVERDIS etc. (29) also discussed about fuzzy soft morphology. These filters are less sensitive to image distortions and to small variations in the shape of the objects. Fuzzy soft morphology performs better in impulse noise removal, compared to standard morphological op’s. Fuzzy soft morphology extended to edge detection also. Soft morphological filters have entered to recursive environment also. SHIH, F. Y. & PADMAJA. P (22) PERTTI KOIVISTO etc. (25) PEI, S. etc. (26) discussed about recursive soft morphology in various contexts and environments. In a research paper (22), the authors discussed about properties up to some extent. But, elimination of noise, as well as, detail preservation are opposite characteristics, up to some extent. A strong smoothing filter may not preserve details. But, in soft morphology, a balanced solutions maybe obtained, which will preserve details as well as suppress noise, due to flexibility in the definition of soft morphology. It is discussed by KOI VISTO, P. etc. (23).

Statistical soft morphological op’s are new type of op’s, which possess two types of advantages. These have properties of dealing with shape for shape preservation, due to soft morphological characteristics as well as noise cleaning properties due to statistical approach in those statistical soft morphological ops. It is introduced by STRINGA, E, etc (24). Like above methodologies, recursive order – statistic soft morphological filters/ op’s, [ROSSM] balance two types of parameters. One is noise reduction. The other os detail/edge preservation. (25).

PEI, S. etc. (26) discussed these techniques and showed with examples that, these filters perform better compared to other filters like morphological filters, soft morphological filters, order–statistic soft morphological filters. A way of implementing, soft morphological op’s, is discussed by LIPEND WANG etc. (27), based on graphic processing unit [GPU], reduces computing time. Statistical soft morphology is extension to soft morphology, discussed by REGAZZONI, C.S. etc. (28), having advantages, compared to , soft morphological op’s, (in image smoothening such as speckle noise handling, processing remote sensing images).

ZHENG MINGJIE etc. (30) developed directional S.E.’s for speckle noise reduction on SAR images. KOI VISTO, P; etc. (31) concentrated on detail preservation while smoothening. ZHEN JI etc. (32), designed soft morphological filter for reducing periodic noise. These results are compared with other spatial domain as well as frequency domain filters techniques. ZHEN JI etc. (33), in another research paper, discussed about periodic noise reduction, by soft morphological filters, in another way [another algorithm],MARSHALL, S etc. (34), used soft morphological filters for elimination of disturbance, caused by solar cosmic rays, in the images obtained by astronomy base. [Solar]. In the same was smoothening, detail preservation filters basing on soft morphology are discussed in 19,29,22,23,24,25,28, etc. papers, introducing extensions of soft morphological filters like statistical soft morphological filters, recursive soft morphological filters etc.

M. Varda Voulia, I. Andreadis and Ph. Tsali Des discussed about soft morphological points in colour image and colour processing point of view,( 19 ) Recursive filters have entered in to mathematical morphology, and recursive morphology has emerged. Again it has entered in to soft morphology, and Recursive soft mathematical morphology has emerged. It is expanded by Shih, F. Y.; Padmaja Puttagunta in paper (22 ). In 23 Koivisto, P; Huttunen; H. Kuosmanen, P discussed soft morphology in stastical probability angle.

Stringa, E; Regazzoni, C. S. have discussed (24) about signal restoration by statistical soft morphological methods.

In ( 25) Pertti Koivisto; Antti Nie Misto discussed about recursive soft morphological filters. Shih, F. Y.; Padmaja Puttagunta also discussed recursive soft morphological filters(22) So many researchers entered in to edge detection using soft morphological op’s. HUANG FENG – GANG; etc. (35) discussed the role of soft morphological ops in edge detection, in noisy environment. ZHANG YING etc. (36) discussed about edge detection. They used PSO [Particle Swarm Optimization method] to choose best edge detection method, suitable to the environment of the image. SONG XIN LUO JUN etc. (37) discussed a method, which minimizes noise, preserve details & detects edges.
XIAOXIN GUO etc, (38), discussed a new type of filter. They integrated soft morphology, laplacian operator as well as, nature of adaptivity. They designed, adaptive soft morphological laplacian filter, for smoothening as well as edge detection. The nature of adaptively is achieved by, employing, 4 directional structuring elements.

Empirical mode decomposition [EMD] is a new concept in the field of signal processing. The technique, extended to analyze two dimensional data is known as bi dimensional EMD. [BEMD].

XIAOFEI YAN etc (39) proposed an edge detection method, integrating BEMD and soft morphology.

WANG TAO etc. (40) discussed an important concept they have applied soft multi – scale operations for edge enhancement in noisy environment

ST RINGA, E. etc. (41) proposed algorithm, for reconstruction of image, using soft morphology and Bayesian process and applied on SAR images.

HAMID, M.S. (42) designed multidimensional soft morphological filters in gray scale environment, using genetic algorithm for optimization for restoration. DONG YAN – ZHI etc. (43) discussed segmentation in soft morphological environment.

TANLIU. Etc. (44) discussed soft morphology and top – hat tr and SPRT – PMHT for identification of targets which are small in infrared environmental images.

In some applications, like character identification, noise will be generated after shape decomposition (using morphological methods). But soft mathematical morphological methods will function excellently in this environment. It is discussed by N. SANTHI & Dr. K. RAMAR (45).

In (46, 47, 48) the discussion is done in Sodar application.

Here some of the techniques are used for signal enhancement.

In 49,50,51,52,53,54,55,56,59,61 the equalities are discussed among various soft morphological operations.

In 49 ,50,53,54 ,55,56 the equalities are discussed in iterative environment.

In 51,52,59,61 THE EQUALITIES ARE DISCUSSED IN NON ITERATIVE ENVIRONMENT.

In 57,58,60,62,63,64,65,66,67 THE DUALITIES ARE DISCUSSED.

In (49) equality is discussed in iterative environment in between soft dilation and soft close in multi scale environment. It is a paper on soft morphology in iterative environment.

In (50) equality is discussed in iterative environment in between soft erosion and soft close in multi scale environment. It is a paper on soft morphological environment. Discussion is done in multi scale environment.

In (51) equality is discussed in iterative environment in between soft erosion, soft dilation and soft close in multi scale environment. It is again a paper in multi scale environment discussed in soft math morphology.

In (52) equality is discussed in iterative environment in between soft dilation and soft open in multi scale environment.

In (53) equality is discussed in iterative environment in between soft dilation and soft open in multi scale environment . In this paper example is also given ,with the help of tables.

In (54) equality is discussed in iterative environment in between soft erosion and soft open in multi scale environment .This concept is discussed with the help of an example.

In (55) equality is discussed in iterative environment in between soft open and soft close in multi scale environment . In this paper the corresponding equations are given elaborately.

In (50) equality is discussed in iterative environment in between soft erosion and soft dilation in multi scale environment .This is discussed elaborative with examples.

In (57) soft dilation is discussed in multiple duals angle in multi scale dimension. These two duals are given for each and every structuring element and for each threshold and general case is also discussed.

In (58) soft erosion is discussed in multiple duals angle in multi scale dimension. These two duals are given for each and every structuring element and for each threshold and general case is also discussed.

In (59) equality is discussed in multi scale environment in between soft open and soft close. The equations are derived and shown clearly.

In (60) soft erosion is discussed in duals angle in multi scale dimension. In this paper the derivations are given clearly and duality is also explained clearly with the help of diagrams.

In (61) equality is discussed in multi scale environment in between soft erosion and soft dilation. In this paper it is treated with good example.

In (62) soft dilation is discussed in duals angle in multi scale dimension. In this paper the derivations are given clearly and duality is also explained clearly with the help of diagrams.

In (63) soft open and soft close is discussed in duals angle in multi scale dimension. In this paper the equations are explained with good example.
In (64) soft open is discussed in duals angle in multi scale dimension. In this paper the derivations are given clearly. The derivations are given for each case.

In (65) soft erosion is discussed in duals angle in multi scale dimension. In this paper the derivation part is given clearly.

In (66) soft close is discussed in duals angle in multi scale dimension. In this paper also derivations are given excellently.

In (67) soft dilation is discussed in duals angle in multi scale dimension. This paper is also filled up with full derivation part.

6 CONCLUSIONS:

In this paper the author discussed about the reason of emerging of soft mathematical morphology. He discussed the differences in between mathematical morphology and soft mathematical morphology. The papers related to applications are discussed in detail. The papers related to properties are also discussed. Some of the papers having same level of operations (like soft morphology which is extension to mathematical morphology) like recursive morphology are also discussed. Some of the papers are discussed dealing with properties like equality, duality are also discussed. Some of the papers dealing with applications of soft mathematical morphology are also discussed. It will create interest in this area to the researchers and they try to develop this area.

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