Intuitionistic Fuzzy Neutrosophic Soft Topological Spaces

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Abstract: In this paper, we introduce a new notion of fuzzy neutrosophic soft set and to study some basic operation and results in fuzzy neutrosophic soft spaces. Further we construct a topology on an intuitionistic fuzzy neutrosophic soft set. The concepts of intuitionistic fuzzy neutrosophic soft closure, intuitionistic fuzzy neutrosophic soft interior, intuitionistic fuzzy neutrosophic soft exterior, intuitionistic fuzzy neutrosophic soft boundary are introduced and some of its properties are studied.

Keywords: soft set, fuzzy soft set, neutrosophic set, neutrosophic soft set, intuitionistic fuzzy neutrosophic soft set, fuzzy neutrosophic soft topological space.

1. INTRODUCTION

The fuzzy set was introduced by Zadeh [13] in 1965 where each element had a degree of membership. In 1999, Molodtsov [4], initiated the novel concept of soft set theory, which was a completely new approach for modeling uncertainty and had a rich potential for application in several directions. This so-called soft set theory is free from the difficulties affecting existing methods. The intuitionistic fuzzy set (IFS for short) on a universe X was introduced by K. Atanassov [1] in 1983 as a generalization of fuzzy set, where besides the degree of membership and the degree of non-membership of each element. The concept of Neutrosophic set was introduced by F. Smarandache [10] which is a mathematical tool for handling problems involving imprecision, indeterminacy and inconsistent data. Pabitra Kumar Maji [7] had combined the Neutrosophic set with soft sets and introduced a new mathematical model ‘Neutrosophic soft set’. Chang [4] introduced the notion of fuzzy topology and also studied some of its basic properties. Shabir and Naz [10] introduced the notion of the soft topology and studied some basic concepts such as soft interior, soft closure and soft sub base. Maji [7] defined the notion of neutrosophic soft set. Arockiarani et al. [1] defined the notion of fuzzy neutrosophic set and fuzzy neutrosophic soft set.

2. PRELIMINARIES

Definition 2.1
Let U be the initial universe set and E be a set of parameters. Let P(U) denote the power set of U. Consider a non-empty set A, A ⊆ E. A pair (F, A) is called a soft set over U, where F is a mapping given by F: A → P(U).

Definition 2.2
Let I^X denote the set of all fuzzy sets on X and A ⊆ E. A pair (f, A) is called a fuzzy soft set over X, where f is a mapping from A into I^X. That is, for each a ∈ A, f(a) = f_a : X → I, is a fuzzy set on X.

Definition 2.3
A neutrosophic set A on the universe of discourse X is defined as

\[ A = \{ (x; T_a(x), I_a(x), F_a(x)), x \in X \} \]

where the functions

\[ T_a, I_a, F_a : X \to [0,1] \]

define respectively the degree of membership, the degree of indeterminacy and the degree of non-membership of the elements to the set A with the condition

\[ 0 \leq T_a(x) + I_a(x) + F_a(x) \leq 3^+ \]

Definition 2.4
Let U be the universe set and E be a set of parameters. Consider a non-empty set A, A ⊆ E. Let P(U) denote the set of all neutrosophic set of U. The collection (F, A) is termed to be the soft neutrosophic set over U, where F is a mapping given by

\[ F : A \to P(U) \]

Definition 2.5
Let U be an initial universe set. Let E be a set of parameters and a non-empty set A ⊆ E. Let IFN(U) denotes the set of all intuitionistic fuzzy neutrosophic sets of U. The pair (F, A) is called intuitionistic fuzzy neutrosophic soft set (in short IFNSS) over U, where F is a mapping given by

\[ F : A \to IFN(U) \]

and IFNSS(F, A) is denoted as \( \hat{F}_A \).
3. FUZZY NEUTROSOPHIC SOFT TOPOLOGICAL SPACES.

Definition 3.1
A fuzzy neutrosophic set \(A\) on the universe of discourse \(X\) is defined as
\[
A = \{(x; T_A(x), I_A(x), F_A(x)), x \in X\}
\]
Where \(T, I, F; X \rightarrow [0, 1]\) and
\[0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.

Theorem 3.2
Let \((F, A), (G, A), (H, A), (S, A) \in FNS\). Then the following are true if
(i) \((G, A) \subseteq (F, A) \iff (F, A) \subseteq (G, A)^C\).
(ii) \((F, A) \subseteq (G, A) \iff (G, A)^C \subseteq (F, A)^C\).

Proof
Given that, \((F, A), (G, A), (H, A), (S, A) \in FNS\).

To prove:
(i) \((F, A) \subseteq (G, A) \iff (F, A) \subseteq (G, A)\).
(ii) \((F, A) \subseteq (G, A) \iff (G, A)^C \subseteq (F, A)^C\).

Proof of (i)
Suppose that \((F, A) \subseteq (G, A)\).
Then \(F(e) \subseteq G(e)\) for all \(e \in A\).
So \(F(e) \subseteq U \setminus G(e) = G^C(e)\) for all \(e \in A\).
Therefore we have,
\[
(F, A) \subseteq (G, A)^C \iff \text{Hence (i) is proved.}
\]

Proof of (ii)
\[
(F, A) \subseteq (G, A) \iff \text{Hence (ii) is proved}
\]

Theorem 3.3
(i) \((F, A) \subseteq (\phi, B) = (\phi, A \cap B)\).
(ii) \((F, A) \subseteq (U, B) = (F, A \cap B)\).

Proof
Proof of (i)
We have for \((F, A)\)
\[
F(e) = \left\{(x; T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) \right\} \forall e \in A
\]
Let \((\phi, B) = (G, B)\) then
\[
G(e) = \{(x, 0, 0,1): x \in U\} \forall e \in B
\]
Let \((F, A) \subseteq (\phi, B) = (F, A) \subseteq (G, B) = (H, C)
Where \(C = A \cap B\) and \(\forall e \in C\)
\[
H(e) = \{(x; \min\left(T_{F(e)}(x), T_{G(e)}(x)\right)), \forall e \in C
\]

\[
\min\left(I_{F(e)}(x), I_{F(e)}(x)\right),
\]
\[
\max\left\{(F_{F(e)}(x), F_{G(e)}(x))\right\}; x \in U\}
\]
\[
= \{(x, \min(T_{F(e)}(x), 0), \min(I_{F(e)}(x), 0),
\]
\[
\max(F_{F(e)}(x), 1)): x \in U\}
\]
\[
= \{(x, 0, 0,1); x \in U\}
\]
\[
= (G, B) = (\phi, B)
\]
Thus, \((F, A) \subseteq (\phi, B) = (\phi, B)\)
Hence (i) proved

Similarly, we can prove (ii).

Definition 3.4
Let \((F, A)\) be \(FNS\) set on \((U, E)\) and \(\tau\) be a collection of fuzzy neutrosophic soft subsets of \((F, A)\). \((F, A)\) is called fuzzy neutrosophic soft topology (FNST) if the following conditions hold
(i) \(\phi, U \in \tau\)
(ii) \(F \cap E \subseteq \tau\) implies \(F \subseteq \tau\).
(iii) \(\{F_E\}_\alpha \subseteq \tau\) implies \(\bigcup \{F_E\}_\alpha \subseteq \tau\).

The triplet \((U, \tau, E)\) is called an fuzzy neutrosophic soft topological space (FNS\(S\)) over \(U\). Every member of \(\tau\) is called a fuzzy neutrosophic soft open set in \(U\).

\(F_E\) is called an fuzzy neutrosophic soft closed set in \(U\) if \(F_E \subseteq \tau\), where
\[
\tau = \{F_E: F \subseteq \tau\}.
\]

Theorem 3.5
Let \((U, \tau_1, E)\) and \((U, \tau_2, E)\) be two fuzzy neutrosophic soft topological spaces. Denote \(\tau = \{F_E: F \subseteq \tau_1\} \cap \tau_2\). Then \(\tau_1 \cap \tau_2\) is an \(FNS\) on \(U\).

Proof
Given that, \((U, \tau_1, E)\) and \((U, \tau_2, E)\) be two fuzzy neutrosophic soft topological spaces. Denote \(\tau = \{F_E: F \subseteq \tau_1\} \cap \tau_2\). Then \(\tau_1 \cap \tau_2\) is an \(FNS\) on \(U\).

To prove
\(\tau_1 \cap \tau_2\) is an \(FNS\) on \(U\).

Obviously, \(\phi, U \in \tau_1 \cap \tau_2\).
Let \(F_E, G_E \in \tau_1 \cap \tau_2\).
Then \(F_E, G_E \in \tau_1\) and \(F_E, G_E \in \tau_2\).
\(\tau_1\) and \(\tau_2\) are two \(FNS\)'s on \(U\).
Then \( F_\eta \cap G_\epsilon \in \tau_1 \) and \( F_\eta \cap G_\epsilon \in \tau_2 \).

Hence \( F_\eta \cap G_\epsilon \in \tau_1 \cap \tau_2 \).

Let \( \{ (F_\alpha)_{\epsilon} : \alpha \in \Gamma \} \subseteq \tau_1 \cap \tau_2 \).

Then \( (F_\alpha)_{\epsilon} \in \tau_1 \) and \( (F_\alpha)_{\epsilon} \in \tau_2 \) for any \( \alpha \in \Gamma \).

Since, \( \tau_1 \) and \( \tau_2 \) are two FNST’s on \( U \).

\[ \bar{\bigcup \left\{ (F_\alpha)_{\epsilon} : \alpha \in \Gamma \right\}} \in \tau_1 \cup \tau_2 \]

Thus, \( \bigcup \left\{ (F_\alpha)_{\epsilon} : \alpha \in \Gamma \right\} \in \tau_1 \cap \tau_2 \).

Let \( \tau_1 \) and \( \tau_2 \) are two FNSTS on \( U \).

Denote, \( \tau_1 \cup \tau_2 = \{ F_\eta \cup G_\epsilon : F_\eta \in \tau_1 \) and \( G_\epsilon \in \tau_2 \} \).

\[ \tau_1 \cap \tau_2 = \{ F_\eta \cap G_\epsilon : F_\eta \in \tau_1 \text{ and } G_\epsilon \in \tau_2 \} . \]

Therefore, \( \tau_1 \cap \tau_2 \) is an FNST on \( U \).

4 ON INTUITIONISTIC FUZZY SOFT TOPOLOGICAL SPACES.

Definition 4.1

Let \( \tau \subset IFSS(X, E) \) be the collection of intuitionistic fuzzy soft sets over \( X \), then \( \tau \) is said to be an intuitionistic fuzzy soft topology on \( X \) if

(i) \( \phi, \bar{\tau} \) belong to \( \tau \).
(ii) The union of any number of intuitionistic fuzzy soft set in \( \tau \) belongs to \( \tau \).
(iii) The intersection of any two intuitionistic fuzzy soft sets \( \tau \) belongs to \( \tau \).

The triple \( (X, \tau, E) \) is called an intuitionistic fuzzy soft topological space over \( X \). If \( (F, G, E) \in \tau \), then the intuitionistic fuzzy soft sets \( (F, G, E) \) is said to be intuitionistic fuzzy soft open set.

Theorem 4.2

Let \( (X, \tau, E) \) be an intuitionistic fuzzy soft topological space over \( X \) and \( (F, G, E) \in \tau \). Then the collection \( \tau_1 = \{ (F_\eta)_{\epsilon} \}_{\epsilon \in \Sigma} \) and \( \tau_2 = \{ (G_\epsilon)_{\epsilon} : \epsilon \in \Sigma \} \) defines a fuzzy soft topology on \( X \).

Proof

Given that, \( (X, \tau, E) \) be an intuitionistic fuzzy soft topological space over \( X \) and \( \tau = \{ (F_\eta)_{\epsilon} \}_{\epsilon \in \Sigma} \).

To prove:

The collection \( \tau_1 = \{ (F_\eta)_{\epsilon} \}_{\epsilon \in \Sigma} \) and \( \tau_2 = \{ (G_\epsilon)_{\epsilon} : \epsilon \in \Sigma \} \) defines a fuzzy soft topology on \( X \).
Proof

Given that, \((X, \tau, E)\) be an intuitionistic fuzzy soft topological space over \(X\), \((F_1, G_1, E)\) and \((F_2, G_2, E)\) are intuitionistic fuzzy soft sets over \(X\).

To prove:

(i) \((F_1, G_1, E)\) is an intuitionistic fuzzy soft closed set if and only if \((F_1, G_1, E) = (F_1, G_1, E)\).

(ii) \((F_1, G_1, E) \subset (F_2, G_2, E)\) if only \((F_1, G_1, E) \subset (F_2, G_2, E)\).

(iii) \((F_1, G_1, E) \subset (F_2, G_2, E)\) implies \((F_1, G_1, E) \subset (F_2, G_2, E)\).

To prove (i)

If \((F_1, G_1, E)\) is an intuitionistic fuzzy soft closed set over \(X\).

Then \((F_1, G_1, E)\) is itself an intuitionistic fuzzy soft closed set over \(X\) which contains \((F_1, G_1, E)\).

So \((F_1, G_1, E)\) is the smallest intuitionistic fuzzy soft closed set containing \((F_1, G_1, E)\) and

\[(F_1, G_1, E) = (F_1, G_1, E)\].

Hence (i) is proved

To prove (ii)

Since \((F_1, G_1, E)\) is an intuitionistic fuzzy soft closed set, by using part (i) we get

\[(F_1, G_1, E) = (F_1, G_1, E)\].

Hence (ii) is proved.

To prove (iii)

Suppose that \((F_1, G_1, E) \subset (F_2, G_2, E)\).

Then every intuitionistic fuzzy soft closed super set \((F_2, G_2, E)\) will also contain \((F_1, G_1, E)\).

This means every intuitionistic fuzzy soft closed super set of \((F_2, G_2, E)\) is also an intuitionistic fuzzy soft closed super set of \((F_1, G_1, E)\).

Hence the intersection of intuitionistic fuzzy soft closed super sets of \((F_1, G_1, E)\) is contained in the intersection of intuitionistic fuzzy soft closed super sets of \((F_2, G_2, E)\).

Thus, \((F_1, G_1, E) \subset (F_2, G_2, E)\)

Hence (iii) is proved.

Definition 4.5

Let \((X, \tau, E)\) be an intuitionistic fuzzy soft topological space over \(X\) and \((F, G, E)\) be an intuitionistic fuzzy soft set over \(X\). Define the interior of \((F, G, E)\) as the join of all the intuitionistic fuzzy soft open subsets contained in \((F, G, E)\) and is denoted by \((F, G, E)^\star\).

Definition 4.6

Let \((X, \tau, E)\) be an intuitionistic fuzzy soft topological space over \(X\) and \((F, G, E)\) be an intuitionistic fuzzy soft set over \(X\). Then the associated interior of \((F, G, E)\) is an intuitionistic fuzzy soft set over \(X\), denoted by \((F^\#, G^\#, E)\) and is defined as

\[
(F^\#, G^\#)(e) = \left( \bigvee_s F_s(e), \bigwedge_s G_s(e) \right)
\]

where \((F_s, G_s, E) \in \tau\) and

\[(F_s(e), G_s(e)) \leq (F(e), G(e)).\]

Theorem 4.7

Let \((X, \tau, E)\) be an intuitionistic fuzzy soft topological space over \(X\) and \((F, G, E)\) be an intuitionistic fuzzy soft set over \(X\). Then \((F, G, E)^\dagger = ((F, G, E)^\dagger)^\dagger\).

Proof

Given that, \((X, \tau, E)\) be an intuitionistic fuzzy soft topological space over \(X\) and \((F, G, E)\) be an intuitionistic fuzzy soft set over \(X\).

To prove

\[(F, G, E)^\dagger = ((F, G, E)^\dagger)^\dagger\].

Let \((F, G, E)\) be an intuitionistic fuzzy soft set.

We show that

\[(F, G, E)^\dagger = ((F, G, E)^\dagger)^\dagger\].

We have

\[
((F, G, E)^\dagger)^\dagger = \left( \bigvee_{(H, Q, E) \in (F, G, E)} (H, Q, E) \right)
\]

\[
= \bigwedge_{(H, Q, E) \in (F, G, E)} (H, Q, E)^\dagger
\]
\[ (F, G, E)^* = (F, G, E) \]

Hence \((F, G, E)^*\) is obtained.

Hence the proof

**SINTUITIONALIST FUZZYNEUTROSOPIHC SOFT TOPOLOGICAL SPACES**

**Definition 5.1**

Let \(U\) be an initial universe set and \(E\) be a set of parameters. An intuitionistic fuzzy neutrosophic soft topology (in short IFNST) \(\tau\) on \(U\) is a family of intuitionistic fuzzy neutrosophic soft sets over \(U\) satisfying the following properties

(i) \(\emptyset, I_x \in \tau\)

(ii) If \(F_A, G_B \in \tau\) Then \(F_A \cap G_B \in \tau\)

(iii) If \(F_{A_i} \in \tau\), \(\forall i \in I\), be any arbitrary index set, then \(\cup \{F_{A_i}\} \in \tau\).

The triplet \((U, E, \tau)\) is said to be an intuitionistic fuzzy neutrosophic soft topological spaces (in short IFNSTS).

**Definition 5.2**

If \(\tau\) is an intuitionistic fuzzy neutrosophic soft topology on \(U\), the triplet \((U, E, \tau)\) is said to be an intuitionistic fuzzy neutrosophic soft topological spaces. Each member of \(\tau\) is called an intuitionistic fuzzy neutrosophic soft open set in \((U, E, \tau)\).

An intuitionistic fuzzy neutrosophic soft set is called intuitionistic fuzzy neutrosophic soft closed if its complement is intuitionistic fuzzy neutrosophic soft open.

**Definition 5.3**

Let \((U, E, \tau)\) be an intuitionistic fuzzy neutrosophic soft topological spaces. Let \(F_A\) be IFNST over \(U\). Then an intuitionistic fuzzy neutrosophic soft closure of \(F_A\) denoted by \(cl(F_A)\) and is defined as the intersection of all intuitionistic fuzzy neutrosophic soft closed sets containing \(F_A\).

\[ cl(F_A) = \cap \{G_B; G_B \text{ is an intuitionistic fuzzy neutrosophic soft closed and } F_A \subseteq \overline{G_B}\} \]

It is also clear that \(cl(F_A)\) is IFNS closed and \(F_A \subseteq cl(F_A)\).

**Theorem 5.4**

Let \((U, E, \tau)\) be IFNSTS. Let \(F_A\) and \(G_B\) are two IFNST over \(U\). Then

\[ cl(F_A \cup G_B) = cl(F_A) \cup cl(G_B) \]

**Proof**

Given that, \((U, E, \tau)\) be IFNSTS. \(F_A\) and \(G_B\) are two IFNST over \(U\).

To prove:

\[ cl(F_A \cup G_B) = cl(F_A) \cup cl(G_B) \]

Since,

\[ F_A \cup G_B \subseteq F_A \cup G_B \]

\[ cl(F_A) \cup cl(G_B) \subseteq F_A \cup G_B \]

\[ cl(F_A) \cup cl(G_B) \subseteq cl(F_A \cup G_B) \]

\[ cl(F_A) \cup cl(G_B) = cl(F_A \cup G_B) \]

So,

\[ cl(F_A) \cup cl(G_B) \subseteq cl(F_A \cup G_B) \rightarrow (1) \]

\[ F_A \subseteq cl(F_A) \]

\[ G_B \subseteq cl(G_B) \]

\[ F_A \cup G_B \subseteq F_A \cup G_B \]

\[ cl(F_A) \cup cl(G_B) \subseteq cl(F_A) \cup cl(G_B) \]

\[ cl(F_A) \cup cl(G_B) \text{ is IFNS closed set containing } F_A \cup G_B \text{ over } U. \]

Then

\[ cl(F_A \cup G_B) \subseteq cl(F_A) \cup cl(G_B) \rightarrow (2) \]

From (1) and (2)

\[ cl(F_A \cup G_B) = cl(F_A) \cup cl(G_B) \]

Hence the proof

**Definition 5.5**

Let \((U, E, \tau)\) be an intuitionistic fuzzy neutrosophic soft topological space. Let \(F_A\) be IFNSTS over \(U\). Then the intuitionistic fuzzy neutrosophic soft interior of \(F_A\) denoted by \(int(F_A)\) and is defined as the union of all intuitionistic fuzzy neutrosophic soft open sets containing \(F_A\).

\[ int(F_A) = \cup \{G_B; G_B \text{ is an intuitionistic fuzzy neutrosophic soft open and } F_A \subseteq \overline{G_B}\} \]

It is also clear that \(int(F_A)\) is IFNS open and \(int(F_A) \subseteq F_A\).

**Theorem 5.6**

Let \((U, E, \tau)\) be IFNSTS. Let \(F_A\) and \(G_B\) are two IFNSTS over \(U\). Then

\[ int(F_A \cap G_B) = int(F_A) \cap int(G_B) \]

**Proof**

Given, \((U, E, \tau)\) be IFNSTS and \(F_A\) and \(G_B\) are two IFNSTS over \(U\).

To prove:

\[ int(F_A \cap G_B) = int(F_A) \cap int(G_B) \]

Since,

\[ F_A \cap G_B \subseteq F_A \]

\[ F_A \cap G_B \subseteq G_B \]

\[ int(F_A \cap G_B) \subseteq int(F_A) \]

\[ int(F_A \cap G_B) \subseteq int(G_B) \]

\[ int(F_A \cap G_B) \subseteq int(F_A) \cap int(G_B) \rightarrow (1) \]

By definition,

\[ int(F_A) \subseteq (F_A) \]

\[ int(G_B) \subseteq (G_B) \]

\[ int(F_A \cap G_B) \subseteq int(F_A \cap G_B) \text{ is the largest IFNS open set contained in } (F_A \cap G_B) \]

\[ int(F_A) \cap int(G_B) \subseteq int(F_A \cap G_B) \rightarrow (2) \]

From (1) and (2)

\[ int(F_A \cap G_B) = int(F_A) \cap int(G_B) \]
Hence the proof

5.7 Definition
Let \((U, E, \tau)\) be an intuitionistic fuzzy neutrosophic soft topological space. Let \(\tilde{F}_A\) be IFNSS over \(U\). Then the \textit{intuitionistic fuzzy neutrosophic soft exterior} of \(\tilde{F}_A\) denoted by \(\text{ext}(\tilde{F}_A)\) and is defined as
\[
\text{ext}(\tilde{F}_A) = \text{int}(\tilde{F}_A^C)
\]

Theorem 5.8
Let \((U, E, \tau)\) be IFNSTS. Let \(\tilde{F}_A\) and \(\tilde{G}_B\) are two IFNSS over \(U\). Then
(i) \(\text{ext}(\tilde{F}_A \cup \tilde{G}_B) = \text{ext}(\tilde{F}_A) \cap \text{ext}(\tilde{G}_B)\).
(ii) \(\text{ext}(\tilde{F}_A) \cap \text{ext}(\tilde{G}_B) \subseteq \text{ext}(\tilde{F}_A \cap \tilde{G}_B)\).

Proof
Given that, \((U, E, \tau)\) be IFNSTS and \(\tilde{F}_A\) and \(\tilde{G}_B\) are two IFNSS over \(U\).

To prove:
(i) \(\text{ext}(\tilde{F}_A \cup \tilde{G}_B) = \text{ext}(\tilde{F}_A) \cap \text{ext}(\tilde{G}_B)\).
(ii) \(\text{ext}(\tilde{F}_A) \cap \text{ext}(\tilde{G}_B) \subseteq \text{ext}(\tilde{F}_A \cap \tilde{G}_B)\).

Hence the proof

Definition 5.9
Let \((U, E, \tau)\) be an intuitionistic fuzzy neutrosophic soft topological space. Let \(\tilde{F}_A\) be IFNSS over \(U\). Then the \textit{intuitionistic fuzzy neutrosophic soft boundary} of \(\tilde{F}_A\) denoted by \(\text{bou}(\tilde{F}_A)\) and is defined as
\[
\text{bou}(\tilde{F}_A) = \text{cl}(\tilde{F}_A) \cap \text{cl}(\tilde{F}_A^C)
\]

Theorem 5.10
Let \((U, E, \tau)\) be IFNSTS. Let \(\tilde{F}_A\) be an IFNSS over \(U\). Then
\[
[\text{bou}(\tilde{F}_A)^C] = \text{int}(\tilde{F}_A) \cup \text{int}(\tilde{F}_A^C)
\]
\[
= \text{int}(\tilde{F}_A) \cup \text{ext}(\tilde{F}_A)
\]
\[
\text{bou}(\tilde{F}_A) = \text{cl}(\tilde{F}_A) \cap \text{cl}(\tilde{F}_A^C)
\]
\[
[bou(\tilde{F}_A)]^C = [\text{cl}(\tilde{F}_A)]^C \cup [\text{cl}(\tilde{F}_A^C)]^C
\]
\[
= \text{int}(\tilde{F}_A^C) \cup \text{int}(\tilde{F}_A)
\]
\[
= \text{int}(\tilde{F}_A) \cup \text{ext}(\tilde{F}_A)
\]
Hence the proof

CONCLUSION
In this dissertation, we discuss some important properties of intuitionistic fuzzy soft topological spaces and define the intuitionistic fuzzy soft closure and fuzzy soft interior of an intuitionistic fuzzy soft topological spaces. We have introduced topological structure on fuzzy neutrosophic soft set and characterized some of its properties. Finally, a new space called intuitionistic fuzzy neutrosophic soft topological spaces are introduced.

BIBLIOGRAPHY