Laminar Boundary Layer Flow of Non-Newtonian Power Law Fluid along Porous Wall of Convergent Channel with Suction /Injection.

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Abstract: Laminar Boundary Layer Flow of Non-Newtonian Power Law Fluid along Porous wall of Wall of Convergent Channel with Suction /Injection has been considered. The governing equations of continuity and momentum are transformed into ordinary differential equation using similarity transformations. The equations are solved by using method of successive approximations starting with zeroth approximation. For $n=1$ the results tallies with corresponding results for Newtonian fluids. Various boundary layer parameters and velocity profiles have been drawn for different values of parameter $n$ and suction/injection $f_w$. $[f'_w (0)]^n$ has been calculated for different values of parameter $n$ and suction/injection $f_w$. 

Keywords: - boundary layer, power law fluids, successive approximations, Convergent channel, velocity profiles, Skin friction, Suction/injection.

1. Introduction

Boundary layer flows of viscous, incompressible fluid past semi infinite flat plate were studied by Blasius[1], Howarth[2]. The boundary layer flow of viscous fluid along the wall of convergent channel was first considered by Pohlhausen[3]. Sanyal[4] studied the two dimensional boundary layers along the wall of convergent channel with curved boundaries. The study of compressible boundary layers along the convergent channel was made by Singh[5].

The study of non –Newtonian viscoelastic fluid along the wall of convergent channel was made by Black and Den[6], Nandlal Singh[7], Kapur and Srivastav[8]. Lee and Ames[9] have obtained similarity solutions to boundary layer flow of power law fluids along the wall of convergent channel. The solution of laminar boundary layer flow of power law fluids along the wall of convergent channel has been obtained by B.P.Jadhav and B.B. Waghmode[10]. B.P.Jadhav [11] has obtained boundary layer parameters for Laminar Boundary Layer Flow of Non-Newtonian Power Law Fluid along Wall of Convergent Channel. In this paper we are going to study the problem considered by B.P.Jadhav and B.B. Waghmode[10] for Porous wall of Wall of Convergent Channel with Suction /Injection effects.

2. Mathematical Analysis

Consider a steady, two dimensional flow of an electrically conducting, non-Newtonian power law fluid along the wall of the convergent channel (fig.-1).

The $x$-axis is taken along the direction of flow and $y$-axis normal to it. Along the wall of the channel the potential flow velocity $U(x)$ is given by:

$$U(x) = -\frac{u_1}{x} \quad (u_1 > 0) \quad \text{(1)}$$
The governing boundary layer equations for the power law fluid flow are

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{\partial U}{\partial x} - \gamma \frac{\partial}{\partial y} \left( - \frac{\partial u}{\partial y} \right)^n \]  

----- (2)

\[ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \]  

----- (3)

With boundary conditions

\[ u=0, \; v=v_w \; \text{at} \; y=0 \; \text{and} \; u=U(x), \; v=0 \; \text{as} \; y \to \infty \]  

----- (4)

To solve the boundary layer equations we introduce a stream function \( \Phi(x,y) \) such that

\[ u = \frac{\partial \Phi}{\partial y}, \; v = -\frac{\partial \Phi}{\partial x} \]  

----- (5)

Introducing similarity transformations,

\[ \eta = y \left( \frac{U^{2-n}}{\gamma x} \right)^{\frac{1}{n+1}}, \quad \Phi(\eta) = \left( \gamma x U^{2n-1} \right)^{\frac{1}{n+1}} f(\eta) \]  

----- (6)

Substituting these values in (6) to the equation (2), it reduces to

\[ nf' + \left( \frac{2n-2}{n+1} \right) f'' + 1 - f'^2 = 0 \]  

----- (7)

Subject to boundary conditions,

\[ f(0) = f_w, f'(0) = 0, f'(\infty) = 1 \]  

----- (8)

**Method of Solution:**

To solve the non-linear differential equation (7) under the boundary conditions (8), we use method of successive approximations starting with zeroth approximation.

For zeroth approximation, we assume

\[ f(\eta) = f_w + \eta - \frac{1}{\beta} + \frac{1}{\beta} e^{-\beta \eta} \]  

----- (9)
where β is arbitrary constant to be determined such that for the first approximation \( f'_1(0) = 0 \), i.e. \( \beta \) is real root of the equation

\[
\beta^{n+1} - \frac{2(n-1)\eta}{n(n+1)(n-2)^2} + \frac{4}{n(n+1)(n-2)^2} - \frac{1}{n(n+1)(n-3)} = 0 \quad -----(10)
\]

The different successive approximations can be obtained from

\[
f'^{2}_{0} = \left( \frac{2-2n}{n+1} \right) f'_{0} + \left( \frac{2-n}{n+1} \right) f''_{0} - 1 \quad -----(11)
\]

For the first approximation, we have,

\[
f'_1 = (f''_0)^{1-n} \left[ \left( \frac{2-2n}{n+1} \right) f'_0 + \left( \frac{2-n}{n+1} \right) f''_0 - 1 \right] \quad -----(12)
\]

Integrating (13) with boundary conditions (9), we obtain

\[
f_1(\eta) = (A_1 - A_2\eta - A_3) e^{(n-2)\eta} - A_4 e^{(n-3)\eta} + \eta + C_1 \quad -----(13)
\]

\[
f_1(\eta) = \left[ \frac{(A_1 - A_2\eta - A_3)(n-2)\beta + A_2}{(n-2)^2 \beta^2} \right] e^{(n-2)\beta} + \left[ \frac{A_4}{(n-3)\beta} \right] e^{(n-3)\beta} + \eta + C_1 \quad -----(14)
\]

Where,

\[
A_1 = \frac{4}{n(n+1)(n-2)^2 \beta^{n+1}}, \quad A_2 = \frac{2(n-1)}{n(n+1)(n-2)^2 \beta^n}, \quad A_3 = \frac{2(n-1)\eta}{n(n+1)(n-2)^2 \beta^n}, \quad A_4 = \frac{1}{n(n+1)(n-3)\beta^{n+1}}, \quad C_1 = f_w - \left( \frac{(A_1 - A_2)(n-2)\beta + A_2}{n-2)^2 \beta^2} \right) + \frac{A_4}{(n-3)\beta}.
\]

\[
-----(15)
\]

**Discussions:-**

Values of \( \beta \) can be obtained for various values of power index \( n \) and suction / injection parameter \( f_w \).

For different values of \( \beta \), power law index \( n \), and suction / injection parameter \( f_w \), values of \( f_1(\eta) \), \( f_2(\eta) \) can be obtained. Hence velocity profiles can be drawn for various values of \( n \) and \( f_w \).

For \( n = 1 \) and zero suction, the results tally with the corresponding results for Newtonian fluids obtained by Pohlhausen[3]. For different values of \( n \) and \( f_w \), skin friction coefficient \( c_{f}^{*} = f'_1(0) \) has been obtained.

**BOUNDARY LAYER PARAMETERS:**

1) **Displacement Thickness:-**

The displacement thickness \( \delta_1 \) is given by

\[
\delta_1 = \int_0^\infty (1 - f'_1(y)) \, dy, \quad \delta_1 \frac{1}{x} (Re)^{\frac{1}{n+1}} = \int_0^\infty (1 - f'_1(y)) \, dy
\]

\[
\delta_1 = \frac{(A_1 - A_3)}{(n-2)\beta} + \frac{A_2}{(n-2)^2 \beta^2} - \frac{A_4}{(n-3)\beta} \quad -----(16)
\]

2) **Momentum Thickness:-**

The Momentum thickness \( \delta_2 \) is given by
Boundary layer parameters for various values of flow index $n$ and zero suction/injection have been shown in Table -1. It has been observed that for increase in $n$, the boundary layer parameters $\delta_1$ and $\delta_2$ increases, while local skin friction $c_f^*$ decreases.

The boundary layer parameters for various values of flow index $n$ and zero suction/injection parameter $f_w$ are given by

$$\delta_2 = \int_0^\infty f_1'(1-f_1') \, dy, \quad \delta_2^* = \frac{1}{\Re} \int_0^\infty f_1'(1-f_1') \, d\eta$$

$$\delta_2^* = \frac{\delta_2}{x} \left( \frac{1}{\Re} \right)^{\frac{1}{n+1}} = \frac{2A_1^2 (n-2)\beta + A_1 A_2^2 (n-2)\beta + A_2^2 (n-3)\beta}{(2n-5)\beta}$$

$$\delta_2 = \frac{2A_1 A_3}{(2n-5)\beta} + \frac{A_2}{(n-2)\beta} + \frac{A_4}{(n-3)\beta} - \frac{A_3}{(n-2)\beta}$$

3) Skin-friction coefficient ($c_f^*$):

The Skin-friction coefficient $c_f^*$ is given by

$$c_f^* = \left[ f_1'(0) \right]^n = \left[ (A_1 - A_3)(n-2)\beta - A_2 - A_4 (n-3)\beta \right]^n \quad (18)$$

The Values of boundary layer parameters for various values of flow index $n$ and different values of suction/injection parameter $f_w$ have been shown in Table -2. It is to be noted that the method employed gives no suction/injection effects on the boundary layer parameters and velocity distribution for $n=1$. The ratio of the boundary layer parameters is near to 2 for all values of flow index $n$.

Fig. 1-5 shows velocity profiles for various values of flow index $n$ and different values of suction/injection parameter $f_w$. Flow profiles for various values of flow index $n$ and without suction/injection has been shown in Fig.-6. For fixed $n$, as suction increases both the boundary layer thicknesses and ratio of the boundary layer thicknesses increases, while local skin friction $c_f^*$ decreases for $n<1$. But for $n > 1$ reverse nature occurs. For fixed $n$ when injection increases both the boundary layer thicknesses and ratio of the boundary layer thicknesses decreases, while local skin friction $c_f^*$ increases for $n<1$ but for $n > 1$ reverse nature occurs. For fixed value of suction/injection parameter $f_w$, as $n$ increases there is increase in both the boundary layer thicknesses and ratio of the boundary layer thicknesses, while local skin friction $c_f^*$ decreases. For fixed value of suction/injection parameter $f_w$, as $n$ increases there is decrease in velocity. For fixed $n$, as suction increases, there is decrease in velocity and as injection increases there is increase in velocity. As $n$ increases then flow function $f_1'(n)$ decreases for zero suction/injection.

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<th>$\delta_1$</th>
<th>$\delta_2^*$</th>
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<th>$c_f^* = [f_1'(0)]^n$</th>
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Table -1 : Boundary layer parameters for various values of $n$

Table -2 : Boundary layer parameters for various values of $n$ and Suction/injection $f_w$
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Velocity profiles for power law fluid flow along wall of convergent channel for different values of \( n \) without suction/injection.

![Diagram](image1)

Velocity profiles for power law fluid flow along wall of convergent channel for \( n=0.5 \) and different values of \( n \).

![Diagram](image2)
Fig.-2 : Velocity profiles for power law fluid flow along wall of convergent channel for $n=0.8$ and different values of $\frac{f_w}{\eta}$

Fig.-3 : Velocity profiles for power law fluid flow along wall of convergent channel for $n=1.5$ and different values of $\frac{f_w}{\eta}$

Fig.-4 : Velocity profiles for power law fluid flow along wall of convergent channel for $n=1.8$ and different values of $\frac{f_w}{\eta}$

Fig.-5 : Graph for Flow function $f_1(\eta)$ without suction
Conclusions:-

The values of arbitrary parameter $\beta$ are calculated for different values of flow index $n$ from the equation (10). For different values of $n$, skin friction coefficient $C_f^*$ is obtained; and presented in the table -1. It has been observed that for $n=1$, the results agrees with the results obtained by Pohlhausen for Newtonian fluid.

As $n$ increases the skin-friction coefficient $C_f^*$ decreases. The present method employed gives good agreements with the available results. As $n$ increases the velocity $f_1'(\eta)$ decreases. The velocity profiles are shown in the figure -1-5. Fig-3 shows that as $n$ increases then flow function $f_1(\eta)$ decreases. Flow profiles in fig-3 and velocity profiles in the fig.-1-5 shows the behavior of power law fluids. For fixed $n$, as suction increases both the boundary layer thicknesses and ratio of the boundary layer thicknesses increases, while local skin friction $C_f^*$ decreases for $n<1$. But for $n>1$ reverse nature occurs. For fixed $n$ when injection increases both the boundary layer thicknesses and ratio of the boundary layer thicknesses decreases, while local skin friction $C_f^*$ increases for $n<1$ but for $n>1$ reverse nature occurs. For fixed $n$, as suction increases there is decrease in velocity and as injection increases there is increase in velocity. As $n$ increases then flow function $f_1(\eta)$ decreases for zero suction/injection.

References


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