Solving A Capacitated Vehicle Routing Problem Using Metaheuristic Algorithm

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Abstract: This paper presents research on CVRP using sweep algorithm and nearest neighbour algorithm. It involves a methodology to find a set of optimal routes that achieve the minimum cost and serve scattered customer locations under several constraints such as vehicle capacity, available vehicle, and customer demand. It consists of two main phases, firstly clustering the given nodes depending up on vehicle capacity and customer demand, the objective of clustering is to group the closest nodes locations together by using Sweep Algorithm. Secondly generating a route for each cluster by using the Nearest Neighbor Algorithm. The case study of APSTRC which provide bus services is selected to present this paper.

Keywords: Capacitated Vehicle Routing Problem, Constraints, Sweep algorithm, and Nearest Neighbor algorithm.

1. Introduction

The Vehicle Routing Problem (VRP) is a well known problem studied by researchers in different areas such as Operations Research, Decision Support Systems, and Artificial Intelligence. The VRP deals with distribution of goods from a depot to a set of customers in a given time period by a fleet of vehicles which are operated by a set of drivers who perform movements on an appropriate road network. The VRP plays a vital role in distribution and logistics where the collection of household waste, gasoline delivery trucks, goods distribution, are the most used applications of the classical VRP. The solution of a VRP is a set of minimum cost routes, which satisfy the problem's constraints, and fulfill customers requirements.

The capacitated vehicle routing problem (CVRP) is a generalization of VRP which represents the most elementary version of the vehicle routing problem. In CVRP model, a fleet of vehicles, located at single or multiple depots need to be scheduled to satisfy the customers’ demands, while visiting every customer exactly once and the capacity of the vehicles must not be exceeded at any point in time. The objective is to obtain a solution that either minimizes the number of vehicles and/or total travelling cost (time or distance). The CVRP can be represented as a weighted directed graph $G = (V, A)$ where $V = \{v_o, v_1, v_2, \ldots, v_n\}$ represents the set of the vertices and $A = \{(v_i, v_j)\ldots, i \neq j\}$ represents the set of arcs. The vertex $v_o$ represents the depot and the others represent the clients. For each arc $(v_i, v_j)$ a nonnegative value $d_{ij}$ is associated. This value corresponds to the distance between the vertex $v_i$ and the vertex $v_j$ in terms of cost or time between the two vertices. A demand $q_i$ and time service $\delta_i$ (where $q_o = 0, \delta_o = 0$) are associated with each client’s vertex $v_i$. In this case, the objective is to minimize the total cost of routing and at the same time respect the following constraints: (1) Every client is visited exactly once by exactly one vehicle, (2) all the vehicles' paths/routes start and end at the depot, (3) the total demand of the clients of each path/route should not exceed the capacity of each vehicle.

2. Literature Review

VRP solution methods are in two main categories: exact methods and heuristics. Exact methods such as branch-and-bound, the branch- and-cut and the branch-and-price algorithms have been developed to solve small-scale VRPs models such small CVRP as in Letchford (2006) which need more computation time. Unlike exact methods, heuristics and meta-heuristics do not provide feasible solutions which proved to be optimal in a global sense. Instead, they provide pretty good ones, but in less time.

Heuristics are methods which produce good solutions in practice but do not guarantee optimality. Metaheuristics give better solutions than classical heuristics, but consume more computational time. Some of the most popular Metaheuristics are simulated annealing (SA), deterministic annealing (DA), tabu search (TS), generic algorithms (GA), ant systems (AS), constraint programming, and neural networks (NN).
There are several heuristics proposed to solve the CVRP and its variants in the literature. The authors G. Dantzig and J. Ramser described how the VRP may be considered as a generalization of the travelling salesman problem (TSP).

Augerat, P. (1995) developed Vehicle Routing Problem, in which a fixed fleet of delivery vehicles of uniform capacity must service known customer demands for a single commodity from a common depot at minimum transit cost. The capacity constraints of the integer programming are formulated.

Authors P. Toth and D. Vigo, have done detailed survey on CVRP in their book “The vehicle Routing Problem” that describes both exact and heuristic methods for VRPs up to (2002).

The authors N. Christofides, A. Mingozzi and P. Toth present tree search algorithms for the exact solution of the VRP incorporating lower bounds computed from 1) shortest spanning k-degree centre tree (k-DCT), and 2) q-routes. The final algorithms also include problem reduction and dominance tests. They present computational results for a number of problems de- rived from the literature. The results of these authors show that the bounds derived from the q-routes are superior to those from k-DCT and that VRPs of up to about 25 customers can be solved exactly.

The authors E. Alba and B. Dorronsoro propose a Cellular Genetic Algorithm (CGA) which is a kind of decentralized population based heuristic, which is used for solving CVRP, improving several of the best existing results so far in the literature. Their study shows a high performance in terms of the quality of the solutions found and the number of function evaluations (effort).

Suresh Nanda (2012) found solution to vehicle routing problem with time windows (VRPTW) and the capacitated vehicle routing problem (CVRP) by using heuristics.

Yuanzhi Wang (2013) solved Multi – Depot Vehicle Routing Problem with Cellular Ant Algorithm by using both the evolutionary rule of cellular, graph theory and the characteristics of ant colony optimization.

Justin C. Goodson (2014) developed expected cost of a prior policies for the multi compartment vehicle routing problem with stochastic demands, an extension of the classical vehicle routing problem where customer demands are uncertain and products must be transported in separate partitions

3. METHODOLOGY

A hybrid heuristic approach based on Sweep algorithm and Nearest Neighbor algorithm (SA & NNA) proposed as a methodology to develop a tour plan for optimizing the distance and able to serve all customers with the minimum travelling cost. The first method is clustering and Route generation is second method used to solve CVRP where it consists of two phases. The first phase is to group customers into clusters by using the Sweep algorithm and the second phase is to find the optimal solution (best route path) for each cluster by using a nearest neighbor algorithm.

Sweep algorithm was firstly introduced by Gillett and Miller (1974) as a heuristic algorithm that have been developed for solving vehicle dispatch problems with load and distance constraints for each vehicle. The objective of sweep algorithm is to clustering the given nodes i.e. the customers are grouped into clusters. The sweep algorithm uses the following steps.

1. Locate the depot as the center of the two-dimensional plane.
2. Compute the polar coordinates of each customer with respect to the depot.
3. Start sweeping all customers by increasing polar angle.
4. Assign each customer encompassed by the sweep to the current cluster.
5. Stop the sweep when adding the next customer would violate the maximum vehicle capacity.
6. Create a new cluster by resuming the sweep where the last one left off.
7. Repeat steps 4 – 6, until all customers have been included in a cluster.

The nearest neighbour algorithm (NNA) was first introduced by J. G. Skellam. The work on the NNA was continued by P.J. Clark and F.C. Evans. The NNA method compares the distribution of distances that occur from a data point to its nearest neighbour in a given data set with a randomly distributed data set. The first strategy that has been introduced and used for solving TSP problem was NNA algorithm Clustering:

Clustering phase starts with computing the polar coordinates for each customer location as \((\theta, d_y)\) where \(\theta\) is the polar angle and \(d_y\) is the distance for all nodes with respect to the depot. General formula for calculating polar angle is represented in Equation 1.

\[
\theta = \tan^{-1} \left( \frac{y}{x} \right)
\]  

(1)

Where \(\theta\) is a angular value of a node (customer location) \(X, Y\) is coordinate values of each node. After computing angular value of each as per third
step in clustering stage is to increase the polar angle and ranking the customer locations according to their angles to make customer list that used to group nodes into clusters by joining nodes beginning of the closest node to the depot that has the smallest polar angle. That process is repeated until it satisfies the capacity constrain for the vehicle while the total demands for all nodes in each cluster not exceeding the vehicle capacity.

**Route Generation:**

The route generation stage is aimed to optimize searching method to find the optimal solution that represents the shortest path between all nodes in each cluster. The Nearest Neighbor Algorithm is used as a heuristic method to find the shortest path between each cluster node as a travelling salesman problem.

After nodes being clustered, the Nearest Neighbour Algorithm is performed to reorder nodes on each cluster based on the distance of one node to another to generate the shortest routes. The distance matrix between all nodes in each cluster is created by using Equation 2 for calculating Euclidean distance. A general formula for calculating the Euclidean distance is represented as

\[ d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \]  \hspace{2cm} (2)

Where \( d_{ij} \) is a distance between two nodes

The first node to be linked in each cluster is the one that has the shortest distance to the depot, the next node to be linked is selected by searching a node that is closest to the first node. This procedure is repeated until all nodes in the same cluster are linked as a route. The same procedure is performed for the next all clusters until all routes are generated.

**4. Case Study**

Andhra Pradesh Road Transport Corporation (APSRTC) will take to the destination. All areas of Visakhapatnam are well connected with a good Road Transport network. APSRTC busses start from various City and Rural Bus Depots and help farmers, students, labours, workers and employees with Visakhapatnam. It maintains approximately 105 bus routes all over city and to its surrounding districts. From those bus routes randomly picked 4 bus routes to find optimal routes with reference to a new depot. The objective is to minimize cost by minimizing the total distance traveled.

<table>
<thead>
<tr>
<th>S.NO</th>
<th>Route</th>
<th>Distance (km)</th>
<th>No. of Trips</th>
<th>No. of Buses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Simhachalam hill-RTC complex</td>
<td>22.00</td>
<td>7</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>Chodavaram-Railway station</td>
<td>75.00</td>
<td>7</td>
<td>02</td>
</tr>
<tr>
<td>3</td>
<td>RTC complex-Yelamanchili</td>
<td>68.88</td>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>Sector-11 - RTC complex</td>
<td>30.386</td>
<td>7</td>
<td>03</td>
</tr>
</tbody>
</table>

Route distance is the distance travelled by the buses from the origin to the destination. A trip is a travel completed by a bus from the origin to the destination. The number of trips shown in the table is the maximum number of travels completed in a route. The number of buses represents the total buses provided by the APSRTC to serve the particular routes. Figure 1 shows the Geographical location of nodes in Vishakhapatnam city with respect to NAD. The polar co-ordinates of each location are measured using Google maps. The x-axis and y-axis values of each node is measured with respect to node 0 i.e. NAD since NAD is considered as a depot.

![Figure 1: Geographical Plot of Each Node In Polar Co-Ordinate Form.](image-url)
TABLE 2: Node Name, X, Y Coordinate Value For Each Customer And Their Demands

<table>
<thead>
<tr>
<th>Node</th>
<th>Node Name</th>
<th>X, Y Coord</th>
<th>Demand</th>
<th>Angle</th>
<th>Quadrant</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>6.20</td>
<td>8.00</td>
<td>22</td>
<td>-13.48</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>7.30</td>
<td>6.00</td>
<td>45</td>
<td>-17.64</td>
</tr>
<tr>
<td>3</td>
<td>C</td>
<td>5.00</td>
<td>5.00</td>
<td>45</td>
<td>-18.47</td>
</tr>
<tr>
<td>4</td>
<td>D</td>
<td>3.31</td>
<td>4.00</td>
<td>45</td>
<td>-20.64</td>
</tr>
<tr>
<td>5</td>
<td>E</td>
<td>4.70</td>
<td>5.00</td>
<td>45</td>
<td>-29.31</td>
</tr>
</tbody>
</table>

5. RESULTS AND DISCUSSION

The Sweep Algorithm and Nearest Neighbor Algorithm (SA & NNA) is applied to the case study. After clustering process is performed by using the sweep algorithm, the optimal routes achieved by using a Nearest Neighbor Algorithm. The resulted clusters and their demands are shown in table 4.

Table 3: Input Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total no of vehicles</td>
<td>23</td>
</tr>
<tr>
<td>Max vehicle capacity</td>
<td>420</td>
</tr>
<tr>
<td>Total demand</td>
<td>2548</td>
</tr>
</tbody>
</table>

Table 4: Optimized Routes

<table>
<thead>
<tr>
<th>Cluster No</th>
<th>Routes</th>
<th>Distance(km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6-25-34-4-3-23-5-2-21</td>
<td>8.7376</td>
</tr>
<tr>
<td>2</td>
<td>0-22-36-9-13-17-18-19-20-21</td>
<td>63.79</td>
</tr>
<tr>
<td>3</td>
<td>24-33-32-31-39-28-0-7-8-9-11-10-12-13-14</td>
<td>68.373</td>
</tr>
</tbody>
</table>

The proposed hybrid algorithm’s results were evaluated by Augerat’s Euclidean benchmark datasets A, B, and P of the CVRP. It can be observed that the sweep algorithm is a simple and powerful tool to perform customers clustering stage while using the Nearest Neighbor algorithm in route generation stage is easy to implement and executes quickly. On the other hand, the disadvantage is that it sometimes missing shorter routes which are easily noticed with human insight. The proposed method is implemented to the case study of Visakhapatnam city bus service. In the case study the data is collected from APSRTC, we considered four routes from central bus station and NAD as depot to find the optimized routes and the results are compared with the current routes.

Table 5: Comparison of routes

<table>
<thead>
<tr>
<th>OPTIMIZED ROUTES</th>
<th>CURRENT ROUTES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Route</td>
<td>Distance(km)</td>
</tr>
<tr>
<td>Madhura - RTC Complex</td>
<td>8.7376</td>
</tr>
<tr>
<td>Chodavaram - Sector II</td>
<td>68.373</td>
</tr>
<tr>
<td>Velanarachidi - NAD</td>
<td>61.790</td>
</tr>
<tr>
<td>Simhachalam - Penkutchi</td>
<td>11.954</td>
</tr>
<tr>
<td>Total Distance</td>
<td>152.884</td>
</tr>
</tbody>
</table>

The current routes cover a total distance of 196.266 km in which optimized routes cover 152.884 km of the total distance. Therefore, we observe that the total travelling distance is minimized which in turn reduces the operating costs. We also observe that the route from RTC complex to NAD and Simhachalam to NAD covering a distance of 8.737 and 11.954 respectively are low compared to other routes so if the capacity and demand constraints are adjusted then the no. of routes get decreased.
Therefore, one of the other objectives can be achieved.

5. CONCLUSION

A hybrid algorithm, Sweep Algorithm & Nearest Neighbor Algorithm (SA&NNA) are used for reducing the total cost of the capacitated vehicle routing problem by grouping geographically closest customers' locations together into clusters and then finding out the shortest path of each cluster in this paper. The clustering stage is performed by Sweep algorithm and finding the optimal solution was achieved by the nearest neighbor algorithm. It can be observed that the sweep algorithm is a simple and powerful tool to perform customers clustering stage while using the Nearest Neighbor algorithm in route generation stage is easy to implement and executes quickly. At the same time, the disadvantage is that if sometimes missing shorter routes which are easily noticed with human insight. The proposed hybrid algorithm is needed more enhancement in the route generation stage to achieve better results. For future work, further optimization algorithms can be applied to develop route generation stage.

6. REFERENCES